

Investigation of subthreshold resonances with the Trojan Horse Method

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Abstract. It is pointed out that the Trojan-Horse method is a suitable tool to investigate subthreshold resonances.

TRANSFER REACTIONS AND TROJAN HORSE METHOD

A similarity between cross sections for two-body and closely related three-body reactions under certain kinematical conditions [1] led to the introduction of the Trojan-Horse method [2, 3, 4]. In this indirect approach a two-body reaction

$$A + x \rightarrow C + c \quad (1)$$

that is relevant to nuclear astrophysics is replaced by a reaction

$$A + a \rightarrow C + c + b \quad (2)$$

with three particles in the final state. One assumes that the Trojan horse a is composed predominantly of clusters x and b , i.e. $a = (x + b)$. This reaction can be considered as a special case of a transfer reaction to the continuum. It is studied experimentally under quasi-free scattering conditions, i.e. when the momentum transfer to the spectator b is small. The method was primarily applied to the extraction of the low-energy cross section of reaction (1) that is relevant for astrophysics. However, the method can also be applied to the study of single-particle states in exotic nuclei around the particle threshold. The basic assumptions of the Trojan Horse Method are discussed in detail in [4], see also [5].

It is the purpose of this contribution to study the transition from the bound ($E_{Ax} < 0$) to the unbound ($E_{Ax} > 0$) region. We study the case where there is an open channel $c + C \neq A + x$ at the $E_{Ax} = 0$ threshold. We show that there is a continuous transition. If there is a subthreshold resonance in the $B = A + x$ -system it can be experimentally studied in the $A + (b + x) \rightarrow C + c + b$ reaction.

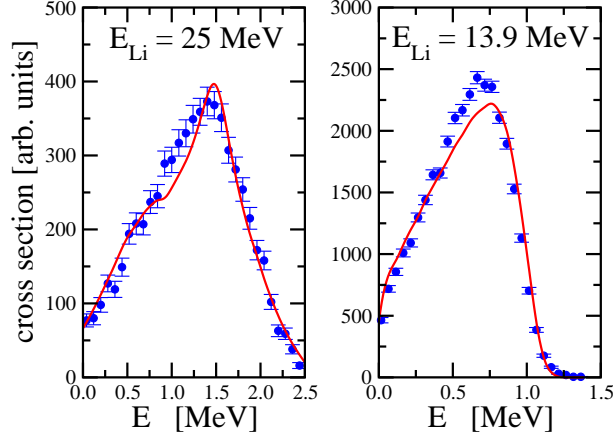


FIGURE 1. The double differential cross section of the Trojan horse reaction $d + {}^6\text{Li} \rightarrow \alpha + {}^3\text{He} + n$ is finite at the $p + {}^6\text{Li}$ relative energy $E = 0$. How does it continue to energies $E < 0$?

CONTINUOUS TRANSITION FROM BOUND TO UNBOUND STATE STRIPPING

For the case where only the elastic channel $A + x$ is open we refer to Ch. 4.2.1 of [6]. Now we study the case where the reaction $x + A \rightarrow C + c$ has a positive Q value, the relative energy E between A and x can be negative as well as positive in the three-body reaction $A + a(= b + x) \rightarrow C + c + b$. As an example we quote the recently studied Trojan horse reaction $d + {}^6\text{Li} \rightarrow \alpha + {}^3\text{He} + n$ [7] applied to the ${}^6\text{Li}(p, \alpha){}^3\text{He}$ two-body reaction (the neutron being the spectator). In this case there are two charged particles in the initial state (${}^6\text{Li} + p$) and the $\alpha + {}^3\text{He}$ -channel is open at the $E = E_{{}^6\text{Li}+p} = 0$ -threshold.

The general question which we want to answer here is how the two regions $E > 0$ and $E < 0$ are related to each other. In fig.1 (fig.7 of [7]) the coincidence yield is plotted as a function of the ${}^6\text{Li}-p$ relative energy E . It is nonzero at zero relative energy. How does the theory [4] (and the experiment) continue to negative relative energies? With this method, subthreshold resonances can be investigated rather directly.

The cross section is a quantity which only exists for $E > 0$. However, a quantity like the S factor (or related to it) can be continued to energies below the threshold. An instructive example is the modified shape function \tilde{S} in Ch. 6 of [8]. In analogy to the astrophysical S factor, where the Coulomb barrier is taken out, the angular momentum barrier is taken out in \tilde{S} . As can be seen from table 3 or 4 of [8] \tilde{S} is well defined for $x^2 < 0$, with the characteristic pole at $x^2 = -1$, corresponding to the binding energy of the $(A + x) = B$ -system.

The inclusive breakup theory of IAV [9, 10] is extended to negative energies $E < 0$ in [6]. We refer the reader to this reference for this approach.

An alternative approach is based directly on the formulation of [4]. The Trojan horse cross section is given in eq.(61) there. The S -matricelement S_{AxCc} can be found from the asymptotics of the radial wave functions, see eq. (23) there. For negative energies $E < 0$

the channel is closed and the outgoing wave function is replaced by an exponentially decaying one. The quantity S_{AxCc} then is no longer an S-matrixelement, but it plays the role of a normalization constant, see also [12, 11].

The Trojan horse amplitude involves the combination $f \propto S_{lc} \cdot J_l^+$ (see eq. (61) of [4]). The threshold behaviour of J^+ in the case of neutrons is given by A.30, for charged particles by eq. 59 of [4]. The energy behaviour of the inelastic $cC \neq Ax$ S-matrixelement is found from the behaviour of the elastic S-matrixelement S_{ll} and unitarity: $|S_{lc}|^2 = 1 - |S_{ll}|^2$ where $l \neq c$. We have $S_{ll} = \exp 2i\delta_l \sim 1 - 2i\delta_l^R - 2\delta_l^I$, where the imaginary part of the phase shift goes like $\delta_l^I \sim (kR)^{2l+1}$ for neutral particles. This leads to $|S_{lc}|^2 \sim (kR)^{2l+1}$. The product $S_{lc} \cdot J_l^+$ is given by $|S_{lc}J_l^+|^2 \sim (kR)^3$ and the cross section $\sigma \propto \frac{1}{k^3} |S_{lc}J_l^+|^2$ tends to a finite constant independent of k . A similar analysis can be done for charged particles.

We now study the smooth change from $E > 0$ to $E < 0$ in a two-channel model with a surface coupling. For the case of $l = 0$ it was studied in Ch. 4.2.3 of [6]. Two radial functions f_1 and f_2 (we assume the same l -value) are coupled by a potential of the type $u_{ij}(r)f_j(r)$. We take $u_{ij} = Q\delta(r - R)$. We are especially interested in the case where there is a resonance just below or above the threshold of channel 1 $\equiv (A + x)$, channel 2 $\equiv (C + c)$ is always open.

In the channel 1 the wave function is $\zeta = Nu_l^+(iqr)$, for $E < 0$ and $\zeta = Su_l^+(kr)$, for $E > 0$. The in- and outgoing wave functions are given by $u_l^\pm = e^{\mp i\sigma_l}(G_l \pm iF_l) \rightarrow \exp(\pm i(kr - \eta \ln(2kr) - \frac{l\pi}{2}))$. For $E < 0$ they correspond to exponentially decreasing and increasing wave functions. The logarithmic derivative in this channel is independent of S or N . For $r \geq R$ the radial wave functions are given in terms of the S-matrix-elements by

$$f_1 = \sqrt{k_2/k_1} \frac{1}{2i} S_{12} u_l^+(k_1 r) \quad (3)$$

and

$$f_2 = \frac{1}{2i} (S_{22} u_l^+(k_2 r) - u_l^-(k_2 r)) \quad (4)$$

The logarithmic matching conditions lead to a 'Sprungbedingung', which determines the S-matrixelements. Denoting the interior logarithmic derivatives ($L \equiv \frac{rf'}{f}$) by L_1 and L_2 we have

$$\frac{Rf_1'(R_{>})}{f_1(R_{>})} \equiv q_l^+(\kappa_1) = L_1 + QR \frac{f_2(R)}{f_1(R)} \quad (5)$$

and

$$\frac{Rf_2'(R_{>})}{f_2(R_{>})} = L_2 + QR \frac{f_1(R)}{f_2(R)} \quad (6)$$

where $\kappa_1 \equiv k_1 R$. Eq. (5) can be solved for f_1 . The LHS is a 'kinematic' quantity, it is given by $q_l^+ \equiv \frac{\kappa u_l^{+'}}{u_l^+}$ and we have

$$f_1(R) = \frac{QR f_2}{q_l^+(\kappa_1) - L_1} \quad (7)$$

For the case of neutral particles (neutrons) we have the following behaviour close to threshold : $\text{Re}q_l^+ = -1 + O(k^2)$ and $\text{Im}q_l^+ \sim (\kappa_1)^{2l+1}/((2l-1)!!)^2 \equiv s_l \equiv \kappa_1 v_l$. The penetration factor s_l is a small number. Charged particles can be treated in an analogous way. Inserting into eq.(9) we obtain an equation for the unknown S-matrixelement S_{22} :

$$\kappa_2 \frac{S_{22} \frac{du_l^+}{d\kappa_2} - \frac{du_l^-}{d\kappa_2}}{S_{22}u_l^+ - u_l^-} = L_2 + \frac{(QR)^2}{q_l^+(\kappa_1) - L_1} \equiv \tilde{L}_2 \quad (8)$$

where $\kappa_2 \equiv k_2 R$. This equation can be solved for S_{22} , we write $S_{22} = \exp(2i\tau_l) S_{22}^{res}$, where τ_l is the hard sphere phase shift. We find

$$S_{22}^{res} = \frac{\tilde{L}_2 - q_l^-(\kappa_2)}{\tilde{L}_2 - q_l^+(\kappa_2)} \quad (9)$$

If channel 1 is closed, $q_l^+(\kappa_1)$ is real, thus \tilde{L}_2 is real and one obtains the form of a single particle resonance with appropriate parameters. The S-matrix is 1×1 and S_{22} is unitary. If channel 1 is open: \tilde{L}_2 is complex and we can bring S_{22}^{res} in a two-channel Breit- Wigner resonance form. S_{12} is then obtained (for a closed or an open channel 1) from eqs. 3 and 7.

Although our model is quite special, the resulting form has a general validity, with ('effective') resonance parameters E_R (position) and partial widths Γ_1, Γ_2 . The total width is $\Gamma = \Gamma_1 + \Gamma_2$. It is a merit of our model that it shows directly how S_{12} is extended to closed channels. It will be interesting to elucidate the relation of the present model to the more general approach of Ref. [13].

The standard Breit-Wigner result is (see e.g.[14])

$$S_{ij} = \exp i(\xi_i + \xi_j) \left(\delta_{ij} - \frac{i\sqrt{\Gamma_i \Gamma_j}}{E - E_R + i\Gamma/2} \right) \quad (10)$$

We now show how to obtain these forms from the coupled channel model. The condition for a resonance at position E_R is $\text{Re}\tilde{L}_2 - \text{Re}q_l^+(E_R) = 0$: This defines E_R . In the resonance energy region we can write $\text{Re}\tilde{L}_2 - \text{Re}q_l^+ = -c_3(E - E_R)$ where c_3 is some constant. The partial widths Γ_1 and Γ_2 are found from $\frac{1}{2}(\Gamma_1 + \Gamma_2) \equiv \frac{-1}{2c_3}(\text{Im}\tilde{L}_2 - \text{Im}q_l^+(\kappa_{2,R}))$ and $\frac{1}{2}(\Gamma_1 - \Gamma_2) \equiv \frac{-1}{2c_3}(\text{Im}\tilde{L}_2 - \text{Im}q_l^-(\kappa_{2,R}))$. We assume that we have a single particle resonance in the uncoupled channel 2, whereas there is no resonance structure in channel 1 (i.e. $\text{Re}q_l^+ - L_1$ is different from zero). The 'bare' resonance condition is $L_2 (= \text{real}) = \text{Re}q_l^+(\kappa_2)$. The coupling induces a shift of the resonance energy.

We have $\text{Im}q_l^+ = -\text{Im}q_l^-(\kappa) = s_l(\kappa)$. Since $s_l(\kappa_1) \ll 1$ we have $\text{Im}\tilde{L}_2 = \frac{-(QR)^2 s_l}{(1+L_1)^2}$. We obtain

$$\Gamma_1 = \frac{2s_l(\kappa_1)(QR)^2}{c_3(l+L_1)^2} \quad (11)$$

and

$$\Gamma_2 = \frac{2s_l(\kappa_2)}{c_3} \quad (12)$$

Thus the Breit-Wigner form of S_{22} is recovered. We see that Γ_1 is strongly energy dependent: it contains the threshold penetration factor $s_l(\kappa_1)$, whereas Γ_2 (and thus also the total width Γ) do not. For $E \leq 0$ the S-matrix consists only of S_{22} , which is unitary. Still, S_{12} which now plays the role of a normalization factor is nonzero.

By direct calculation we can bring S_{12} in the Breit-Wigner form: We write $f_2(\kappa_2) = S_{22}u_l^+ - u_l^- = u_l^-(S_{22}^{res} - 1)$. From eqs. (3,4) and (7) we find

$$S_{12} = \sqrt{k_1/k_2} \frac{-QRu_l^-(\kappa_2)}{u_l^+(\kappa_1)(l+L_1)} \frac{-i\Gamma_2}{E - E_R + \frac{i}{2}(\Gamma_1 + \Gamma_2)} = e^{i(\tau_1 + \tau_2)} \frac{-i\sqrt{\Gamma_1\Gamma_2}}{E - E_R + \frac{i}{2}(\Gamma_1 + \Gamma_2)} \quad (13)$$

We used eqs. (11) and (12), $u_l^\pm = \frac{1}{\sqrt{v_l}} e^{\mp i\tau_l}$ and $s_l \equiv \kappa v_l$ to obtain this result.

This result is valid for $E > 0$. For $E < 0$ there are some simple modifications. The wave number becomes imaginary, $k = iq$. The quantity q_l^\pm is real, u_l^\pm can again be written as $1/\sqrt{v_l} \exp \frac{-i\pi l}{2}$ (for neutral particles). Thus we have $\tau_1 = \frac{i\pi l}{2}$ for $E < 0$. Formally, the penetration factor s_l turns imaginary ($s_l = i(-1)^{2l}(qR)^{2l+1}/((2l-1)!!)^2$), and so does the width Γ_1 , see eq. (11).

For $E_R > 0$ we have a resonance, for $E_R < 0$ a subthreshold resonance, the formulae are valid for both cases. The opening of channel 1 at threshold will induce cusp effects in the elastic $C + c \rightarrow C + c$ scattering, see e.g. [15].

Large $p_{1/2}$ -scattering length in the ^{11}Be system due to a neutron halo state

The electromagnetic dipole strength in ^{11}Be was deduced [16] from high-energy ^{11}Be Coulomb dissociation measurements at GSI [17]. Using a cutoff radius of $R = 2.78$ fm and an inverse bound-state decay length of $q = 0.1486 \text{ fm}^{-1}$ as input parameters we extract an ANC of $C_0 = 0.724(8) \text{ fm}^{-1/2}$ from the fit to the experimental data. The ANC can be converted to a spectroscopic factor of $C^2S = 0.704(15)$ that is consistent with results from other methods. In the lowest order of the effective-range expansion the phase shift δ_l^j in the partial wave with orbital angular momentum l and total angular momentum j is written as $\tan \delta_l^j = -a_l k^{2l+1} = -(xc_l^j \gamma)^{2l+1}$, where $\gamma = qR = 0.4132 < 1$ is the halo expansion parameter and $x = k/q = \sqrt{E/S_n}$. The neutron separation energy is S_n . The parameter c_l^j corresponds to the scattering length $a_l^j = (c_l^j R)^{2l+1}$. We obtain $c_1^{3/2} = -0.41(86, -20)$ and $c_1^{1/2} = 2.77(13, -14)$. The latter is unnaturally large because of the existence of a bound $\frac{1}{2}^-$ state close to the neutron breakup threshold in ^{11}Be .

The connection of the scattering length a_l and the bound state parameter q for $l > 0$ is given by $a_l = \frac{2(2l-1)R^{2l-1}}{q^2(2l+1)!!(2l-1)!!}$, where a square well potential model with a range R was assumed. This is a generalization of the well-known relation $a_0 = 1/q$ for $l = 0$. The $p_{1/2}$ channel in ^{11}Be is an example for the influence of a halo state on the continuum. The binding energy of this state is given by 184 keV, which corresponds to $q = 0.094 \text{ fm}^{-1}$. With $R = 2.78 \text{ fm}$ one has $\gamma^2 = 0.068$. For $l = 1$ one has $a_1 = \frac{2R^3}{3\gamma^2} = 210 \text{ fm}^3$ which

translates into $c_1 = (a_1/R^3)^{1/3} = 2.14$. This compares favourably with the fit value given in table 1 of [16]: $2.77(13, -14)$. The corresponding scattering length is given by $a_1^{1/2} = 457(67, -66) fm^3$. For a further discussion we refer to [16]. The large $p_{1/2}$ -scattering length would also manifest itself in the $^{10}Be(d, p\gamma)^{11}Be$ 'radiative transfer reaction'.

CONCLUSION AND OUTLOOK

The treatment of the continuum is a general problem, which becomes more and more urgent when the dripline is approached. We studied the transition from bound to unbound states as a typical example. In the present analysis this transition is continuous, it is expected that this also shows up in the experimental data. A minireview of the applications of the Trojan horse method to astrophysical reactions can be found in [19]. An extension to stripping into the continuum would be of interest for this and other kinds of reactions. The trojan horse reaction $^{18}F(d, n\alpha)^{15}O$ would be of great interest for the $^{18}F(p, \alpha)^{15}O$ reaction relevant for nova nucleosynthesis [18, 20]. This would also be of interest for SPIRAL2, for example. Another most interesting example would be the radiative α -transfer reaction $^{12}C(^6Li, d\gamma)^{16}O$, where one could also look directly for the subthreshold 1^- and 2^+ states which are crucial for the S-factor of the astrophysically important α -capture reaction $^{12}C(\alpha, \gamma)^{16}O$.

REFERENCES

1. H. Fuchs et al., *Phys. Lett. B*, **37**, 285 (1971).
2. G. Baur, *Phys. Lett. B*, **178**, 135 (1986).
3. S. Typel and H. H. Wolter, *Few Body Systems*, **29**, 75 (2000).
4. S. Typel and G. Baur, *Ann. Phys.*, **305**, 228 (2003).
5. A. M. Mukhamedzhanov, C. Spitaleri, R. E. Tribble, nucl-th/0602001
6. G. Baur, S. Typel nucl-th/0504068, Proceedings of the Workshop on "Reaction Mechanisms for Rare Isotope Beams", Michigan State University, March 9-12, 2005.
7. A. Tumino et al., *Phys. Rev. C*, **67**, 065803 (2003).
8. S. Typel and G. Baur, *Nucl. Phys. A* **759**, 247 (2005), nucl-th/0411069.
9. M. Ichimura, N. Austern, and C. M. Vincent, *Phys. Rev. C*, **32**, 431 (1985).
10. M. Ichimura, *Phys. Rev. C*, **41**, 834 (1990).
11. A. M. Lane and R. G. Thomas *Rev. Mod. Phys.* **30**(1958)257
12. E. P. Wigner, *Phys. Rev.*, **73**, 1002 (1948).
13. A. M. Mukhamedzhanov and R. E. Tribble, *Phys. Rev. C*, **59**, 3418 (1999)
14. C. E. Rolfs and W. S. Rodney, *Cauldrons in the Cosmos* (The University of Chicago Press, Chicago, 1988)
15. R. G. Newton, *Scattering Theory of Waves and Particles*, McGraw-Hill, New York, 1966
16. S. Typel and G. Baur, *Phys. Rev. Lett.*, **93**, 142502 (2004).
17. R. Palit et al., *Phys. Rev. C*, **68**, 034318 (2003).
18. J. Jose and A. Coc *Nuclear Physics News* Vol.15, No.4, 2005 p.17
19. G. Baur and S. Typel nucl-th/0601004, Proceedings of the DAE-BRNS 50th Symposium on Nuclear Physics, Bhabha Atomic Research Centre, Mumbai, India December 12-16, 2005
20. J. C. Blackmon *J. Phys. G* **31** (2005) S1405